Chapter 7
Engineering

7.1 Introduction

Pendulums are an essential component of some engineering structures. Three of these are described in this chapter. These are the Watt steam governor, cable cars, and tension leg platforms.

The *Watt steam governor*, also known as a *centrifugal governor* or *fly ball governor*, and in French régulateur à boules (ball governor) was invented by James Watt to regulate the supply of steam to his steam engines and hence keep the speed reasonably constant, irrespective of the load (Lineham 1914). A Watt steam governor fitted to a Boulton and Watt steam engine ca. 1810 is shown in Fig. 7.1. It is not clear when James Watt started to use his steam governor. Denny (2002) states that it was in 1788, but the Science Museum in London have a Boulton and Watt steam engine, dated 1781, which is fitted with a Watt steam governor. Other applications for what is then known as the *Watt governor* include windmills, lighthouses and telescope drives (Denny 2002; Tobin 2003). Figure 7.2 shows a Watt governor fitted to a corn mill.

Cable cars are widely used in mountainous areas for transporting people and freight up and down mountains. A cable car has an inline set of grooved wheels that run on a *fixed cable* between two stations (Fig. 7.3). A fixed cable is usually supported by pylons between the stations so that it follows the contour of the mountain (Fig. 7.4). A *gondola* is suspended below the wheel set by a girder that is fixed to the gondola, but pivoted to the wheel set. The gondola is therefore the bob of a pendulum. The pendulum must be correctly designed for satisfactory operation of the cable car.

An important design requirement for the *platforms* used offshore for oil and gas exploration and production is to avoid excitation of structural resonances at *wave passing frequencies*. These are typically in the range 0.08–0.4 Hz (Pook and Dover 1989; Patel and Witz 1991). A *fixed platform* is a structure standing on the sea bed. Avoidance of structural resonances in fixed platforms becomes increasingly difficult as the water depth increases, and they are not normally used if the depth is

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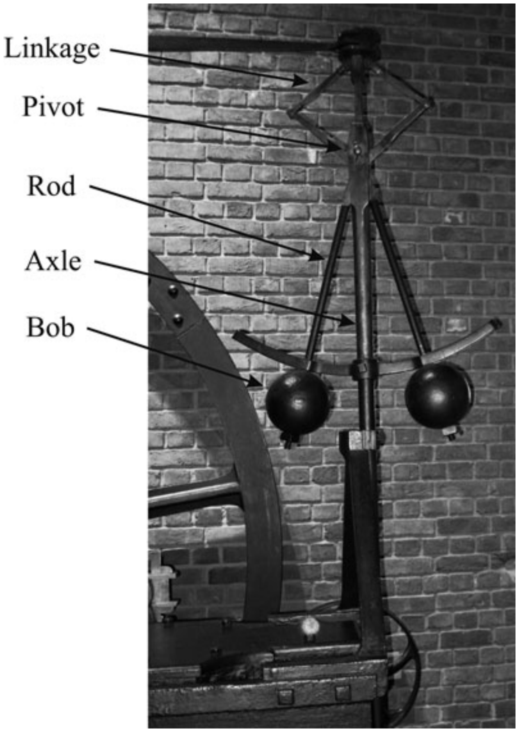

Fig. 7.1 Watt steam governor fitted to a Boulton and Watt steam engine ca. 1810

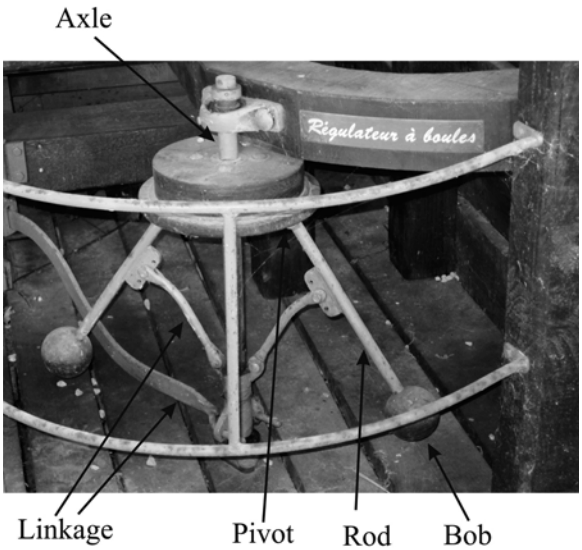

Fig. 7.2 Watt governor fitted to a corn mill

Fig. 7.3 Cable car and station at Aiguille du Midi, France

Fig. 7.4 Cables and pylon for cable cars at Stresa, Italy

greater than about 200 m. For deeper water *tension leg platforms* are now widely used. A tension leg platform is a *floating platform* which is held to the seabed by *tethers*. Tension in the tethers holds the platform below its equilibrium floating level. A tension leg platform is therefore an *inverted pendulum*. A typical tension leg platform is shown schematically in Fig. 7.5. A tension leg platform was first used to produce oil in 1984 (Patel and Witz 1991).

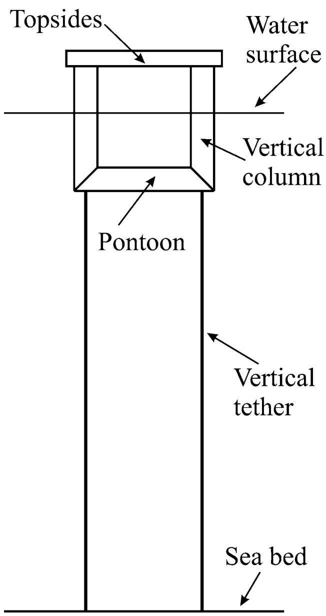

Fig. 7.5 Essential features of a typical tension leg platform

7.2 Watt Steam Governor

Watt steam governors all have the same basic design. There are two identical pendulums, with spherical bobs and light rods attached by a horizontal pivot to a rotating axle driven by the steam engine (Fig. 7.1). The pendulums are real versions of the rotating simple rod pendulum (Fig. 3.7). One of the functions of the associated *linkage* is to ensure that the pendulum angle, θ, shown in Fig. 3.7, is the same for both pendulums. Thus, the governor remains balanced and vibration is avoided. With the governor stationary the pendulum angle is usually about 15° (Fig. 7.1).

The main function of a Watt steam governor, and associated linkage, is to keep the steam engine speed reasonably constant, irrespective of the load (Lineham 1914; Denny 2002). The time of swing, T, for one revolution of the governor is given by Eq. 2.37 where l is the effective length of the pendulums, θ is the pendulum angle and g is the acceleration due to gravity. From Eq. 2.37 (Lineham 1914), the vertical distance of the centre of oscillation of each pendulum below the pivot, l_V (Fig. 3.7) is given by Eq. 3.9 where ω is the angular velocity and is independent of l. Thus, l_V is inversely proportional to ω^2. The linkage transfers the motion of the pendulums to a valve which reduces the supply of steam as the angular velocity increases. Details of the linkage vary widely.

In operation the spherical bobs move out to a pendulum angle, θ, (Fig. 3.7) which depends on the angular velocity of the rotating axle. If the engine speed increases so does the pendulum angle, and the associated linkage reduces the supply of steam to the engine and the engine speed decreases. Conversely, if the engine speed decreases

the supply of steam is increased. In practice, the engine speed is not independent of the load; there is always some decrease in engine speed as the load increases. Hence, the governor needs to be made as sensitive as possible to changes in engine speed. However, if the governor is too sensitive the engine speed oscillates about the desired value, this is known as *hunting*. James Clerk Maxwell's theoretical analysis, published in 1868, showed why hunting occurred, and indicated how it could be avoided. Maxwell's analysis is reproduced by Denny (2002). Maxwell's analysis initiated the discipline now known as *control engineering*.

Before Maxwell's analysis was available Watt steam governor designs had to be optimised by trial and error. In particular, the relationship between the angular velocity of the axle and steam valve opening had to be correct. To achieve this, pendulum pivots were sometimes offset from the rotating axle. The offset can be either positive or negative. A positive offset for an idealised Watt steam governor is shown in Fig. 7.6a. The Watt governor fitted to a corn mill, shown in Fig. 7.2, has a positive offset, but this is not clearly visible in the photograph. A Watt steam governor with negative offset (Fig. 7.6b) is known as *Head's governor* (Lineham 1914).

For a positive offset, X, (Fig. 7.6a) the effective pendulum length, l, is increased by $X/\sin\,\theta$, where θ is the pendulum angle, and there is a *virtual frictionless pivot* above the frictionless pivot. Equation 2.13 for the time of swing, T, for one revolution of the governor becomes

$$T = \frac{1}{2\pi}\sqrt{\frac{l\cos\theta + X\cot\theta}{g}} \tag{7.1}$$

where g is the acceleration due to gravity. Thus, the effect of the positive offset is to change the relationship between the pendulum angle and time of swing. For a

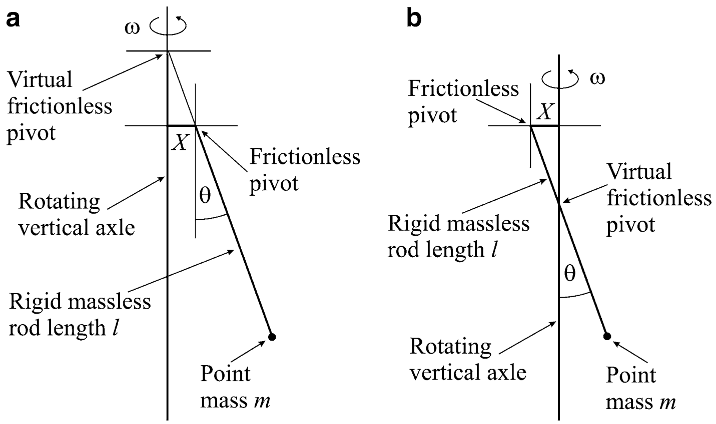

Fig. 7.6 Idealised Watt steam governor pendulums. (**a**) Pivot with positive offset. (**b**) Pivot with negative offset (Head's governor)

given pendulum angle the time of swing is increased, and the relationship between the angular velocity and the vertical distance of the centre of oscillation of each pendulum below the pivot, l_V, is different.

Similarly, for a negative offset (Fig. 7.6b) the virtual frictionless pivot is below the frictionless pivot, and Eq. 7.1 becomes

$$T = \frac{1}{2\pi}\sqrt{\frac{l\cos\theta - X\cot\theta}{g}} \tag{7.2}$$

For a given pendulum angle the time of swing is reduced.

Details of the linkages associated with Watt governors, and Watt steam governors vary widely. Various arrangements were tried in order to optimise the design. In Fig. 7.1 the linkage is above the pivot, and in Fig. 7.2 it is below the pivots.

7.3 Cable Car

Cable cars are widely used in mountainous areas for transporting people and freight up and down mountains. A cable car has an inline set of grooved wheels that run on a *fixed cable* (Fig. 7.7). A *towing cable*, below the fixed cable in the figure, is attached to an *inline wheel set* and moves the cable car along the fixed cable. A fixed cable is usually supported by pylons between the stations so that it follows the contour of the mountain (Fig. 7.4). Cable cars are usually in pairs, attached to the same

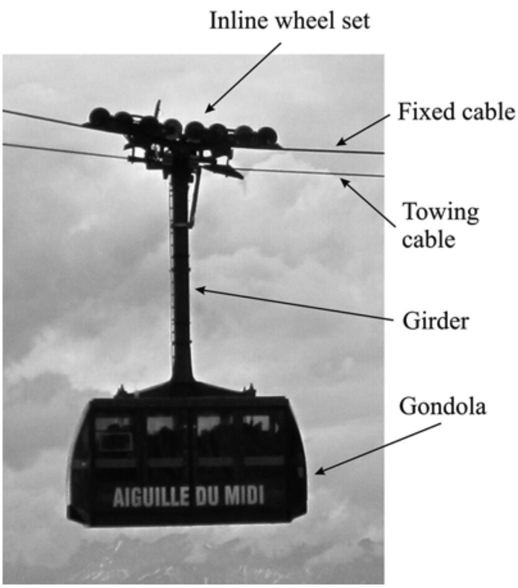

Fig. 7.7 Cable car at Aiguille du Midi, France

towing cable so that the cable car going up the mountain is at least partially balanced by the other cable car coming down the mountain. They pass each other midway. The towing cable is driven by an electric motor in one of the stations. A *gondola* (Figs. 7.3 and 7.7) is suspended below the wheel set by a girder that is fixed to the gondola, but pivoted to the bogies carrying the inline wheel set. The bogies are pivoted so that the wheels follow the curvature of the cable, and can pass smoothly over the abrupt changes in cable curvature at pylons (Fig. 7.4). There is always additional curvature due to the weight of the gondola. This additional curvature is visible in Fig. 7.7. If the gondola were suspended from a single wheel it could oscillate in *front-to-back motion* as viewed in the figure, but the cable curvature due to the inline wheel set means that this does not happen.

The girder connecting the inline wheel set to the gondola is shaped so that it clears the fixed and towing cables, and can pass the fixed cable supports at pylons. The girder is pivoted at the wheel set so that it can swing in *left-right motion*. This means that the gondola remains approximately horizontal irrespective of the inclination of the wheel set, and also irrespective of the distribution of passengers and freight within the gondola. It also means that the gondola and girder form a real version of a simple rod pendulum (Sect. 2.3), and can oscillate in left-right motion. Oscillation in left-right motion with a time of swing of a few seconds is clearly noticeable when travelling in a cable car. One of the reasons that the girder is fairly long is that a long time of swing is more comfortable for passengers than a short time of swing. Another reason is that a long girder means that the pendulum angle, θ, (Fig. 2.2) does not alter much when the distribution of passengers and freight within the gondola changes. This is important for the comfort of passengers, and when passengers are embarking and disembarking.

7.4 Tension Leg Platform

Designs of *tension leg platforms* used for oil and gas exploration and production in waters deeper than about 200 m vary widely (Patel and Witz 1991). The essential features of a typical tension leg platform are shown in Fig. 7.5. There are four cylindrical *pontoons*, arranged in a square, and connected by four surface piercing, vertical, cylindrical *columns* to the *topsides*. The accommodation block, production facilities, etc. are mounted on the topsides. The small water line area of the vertical columns minimises the effect of waves on the platform. There is a vertical *tension leg*, also called a *tether*, at each corner connecting the platform to the sea bed. Each tether is a group of wire ropes. The buoyancy (cf Sect. 4.3) of the pontoons and the immersed part of the vertical columns is greater than that needed to support the weight of the platform. Hence, the platform is held in position by tension in the vertical tethers, and it is an *inverted pendulum*. Specifically, the tension leg platform shown in Fig. 7.5 is an inverted real version of a quadrifilar pendulum (Sect. 3.7).

A tension leg platform has two principal modes of oscillation. These are analogous to modes of oscillation of a quadrifilar pendulum. Amplitudes are small,

and motions approximate to damped simple harmonic motion (Sect. 4.2.1). The first is a *sway mode of oscillation*, in which the platform remains horizontal, and individual points move on the surface of a sphere. This motion is identical to the motion of the point mass, m, of a simple string pendulum (Sect. 2.4). The second is a torsional mode of oscillation in which the platform remains horizontal and rotates about a vertical axis.

For a particular tension leg platform an effective value of the acceleration due to gravity, g_e, can be defined as

$$g_\mathrm{e} = \frac{B}{M} - g \tag{7.3}$$

where B is the buoyancy and M is the mass of the tension leg platform, and g is the acceleration due to gravity. Replacing g by g_e Eq. 2.13 for the time of swing, T, becomes, for the sway mode of oscillation

$$T = 2\pi\sqrt{\frac{l}{g_\mathrm{e}}} \tag{7.4}$$

where l is the length of the tethers. Similarly, for the torsional mode of oscillation Eq. 3.12 becomes

$$T = 2\pi\sqrt{\frac{\kappa^2 l}{g_\mathrm{e} c^2}} \tag{7.5}$$

where κ is the radius of gyration and $2c$ is the diagonal distance between the tethers.

Times of swing, T, for both modes of oscillation are typically 40 s for 120 long m tethers and 80 s for 500 m tethers (Patel and Witz 1991). Corresponding frequencies are 0.025 and 0.0125 Hz. A tension leg platform is driven by surface waves, in which the water surface elevation, and resulting forces on the vertical columns, approximate to narrow band random processes (Pook and Dover 1989). However, wave passing frequencies are typically in the range 0.08–0.4 Hz (Sect. 7.1). Hence, the spectral density function and resonance curves (Figs. 5.5 and 5.6) do not overlap and, as desired, excitation of oscillations does not occur (Sect. 5.4.2).

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